1. Let consider the following equation

\[ \frac{dy}{dx} = \frac{y(x)}{x^2}. \]

- For all \( a, b \in (0, +\infty) \), the equation has a unique solution on \([a, b]\) such that \( y(a) = y_0 \), for all \( y_0 \in (0, +\infty) \) fixed,
- There is a non-zero solution over \( \mathbb{R} \setminus \{0\} \).

(i) TT,
(ii) FT,
(iii) TF,
(iv) FF.

2. Let consider the following equation:

\[ \frac{dy}{dt} = \cos(t) \cdot e^{-y(t)}. \]

- The general solution is given by \( \ln(\sin(t) + C), C \in \mathbb{R} \),
- If \( y(0) = -1 \), the unique solution is defined on all bounded interval of \( \mathbb{R} \).

(i) TT,
(ii) FT,
(iii) TF,
(iv) FF.

3. Let consider the following equation

\[ \frac{2x}{1 + (x^2 + y)^2} + \frac{y'}{1 + (x^2 + y)^2} = 0, \quad y(0) = 0. \]

- There is a function \( H \) such that \( y \) given by \( H(x, y(x)) = H(0, 0) \) is a solution of the equation,
- \( \arctan(x^2 + y) \) is solution of the equation.

(i) TT,
(ii) FT,
(iii) TF,
(iv) FF.
4. What is the general solution of the following equation

\[ y'(t) - \frac{1}{t} y(t) - (y(t))^2 = 0 \]

*Hint: make the substitution \( y(t) = -1/z(t) \).*

(i) \( y(t) = -\frac{2t}{C+t^2}, \ C \in \mathbb{R} \).

(ii) \( y(t) = -\frac{2C}{C_1+t^2}, \ C_1, C_2 \in \mathbb{R} \).

(iii) \( y(t) = -\frac{2C}{t^2}, \ C \in \mathbb{R} \).

(iv) None of the previous choices.

5. Let consider the differential equation

\[ ty'' + (1 - 2t)y' + (t - 1)y = 0 \]

over \((0, +\infty)\).

- \( t \mapsto e^t \) is a solution.
- The general solution is given by \( t \mapsto (\lambda \ln(t) + \mu)e^t, \ \lambda, \mu \in \mathbb{R} \).

(i) TT,

(ii) FT,

(iii) TF,

(iv) FF.

6. Let \( u: (0, +\infty) \longrightarrow \mathbb{R} \) the solution of

\[ u'(t) = \sqrt{u(t) + \sin(t) - \cos(t)}, \]

with \( \lim_{t \to 0^+} u(t) = 0 \). Find \( u \). *Hint: make an judicious substitution.*

(i) \( u(t) = \frac{1}{4} t^2 + \sin(t) \),

(ii) \( u(t) = \frac{1}{4} t^2 C - \sin(t), \ C \in \mathbb{R} \),

(iii) \( u(t) = \frac{1}{4} t^2 - C \sin(t), \ C \in \mathbb{R} \),

(iv) None of the previous choices.

7. Solve the following differential equation:

\[ y'' - 4y' + 4y = 0. \]

The solutions are:

(i) \( y(t) = ae^{2t} + be^{-2t}, \ \text{with} \ a, b \in \mathbb{R} \),

(ii) \( y(t) = (a + bt)e^{2t}, \ \text{with} \ a, b \in \mathbb{R} \),
(iii) \( y(t) = a \cos(2t) + b \sin(2t) \), with \( a, b \in \mathbb{R} \),

(iv) \( y(t) = a \cos(2t)e^{-t} + b \sin(2t)e^{-t} \), with \( a, b \in \mathbb{R} \).

8. Solve the following differential equation :

\[
3y'' + 6y' + 12y = 12t.
\]

The solutions are :

(i) \( y(t) = (a + bt)e^{-t} + t - \frac{1}{2} \), with \( a, b \in \mathbb{R} \),

(ii) \( y(t) = ae^{-t}\cos(t\sqrt{3}) + be^{-t}\sin(t\sqrt{3}) \), with \( a, b \in \mathbb{R} \),

(iii) \( y(t) = ae^{-t}\cos(\frac{2\pi}{3}t) + be^{-t}\sin(\frac{2\pi}{3}t) + t - \frac{1}{2} \), with \( a, b \in \mathbb{R} \),

(iv) \( y(t) = ae^{-t}\cos(t\sqrt{3}) + be^{-t}\sin(t\sqrt{3}) + t - \frac{1}{2} \), with \( a, b \in \mathbb{R} \).

9. Solve the following differential equation :

\[
x^2y'' + xy' - y = x^2.
\]

The solutions are : \textit{Hint : make the substitution} \( z(t) = y(e^t) \).

(i) \( y(x) = ax + \frac{x^2}{3} \), with \( a \in \mathbb{R} \),

(ii) \( y(x) = \frac{a}{2} + \frac{x^2}{3} \), with \( a \in \mathbb{R} \),

(iii) \( y(x) = \frac{a}{2} + bx + \frac{x^2}{3} \), with \( a, b \in \mathbb{R} \),

(iv) None of the previous choices.

10. Solve the following differential equation :

\[
y'' + 4y' + 3y = 0.
\]

The solutions are :

(i) \( y(t) = ae^{-3t} + be^{-t} \), with \( a, b \in \mathbb{R} \),

(ii) \( y(t) = (a + bt)e^{-4t} \), with \( a, b \in \mathbb{R} \),

(iii) \( y(t) = a \cos(-3t) + b \sin(-t) \), with \( a, b \in \mathbb{R} \),

(iv) None of the previous choices.